Mastering the work-conserving properties in the context of the optimization-based approach of GPU timing analysis for real-time systems.

Kostiantyn Berezovskyi

Konstantinos Bletsas

Eduardo Tovar

CISTER - Research Centre in Real-Time & Embedded Computing Systems

isep Instituto Superior de Engenharia do Porto

P.PORTO

Eduardo Tovar
CISTER ISEP/IPP
Porto, Portugal

Konstantinos Bletsas
CISTER ISEP/IPP
Porto, Portugal

Luca Santinelli
Onera
Toulouse, France

Stefan M. Petters
CISTER/INESC-TEC, ISEP
Porto, Portugal

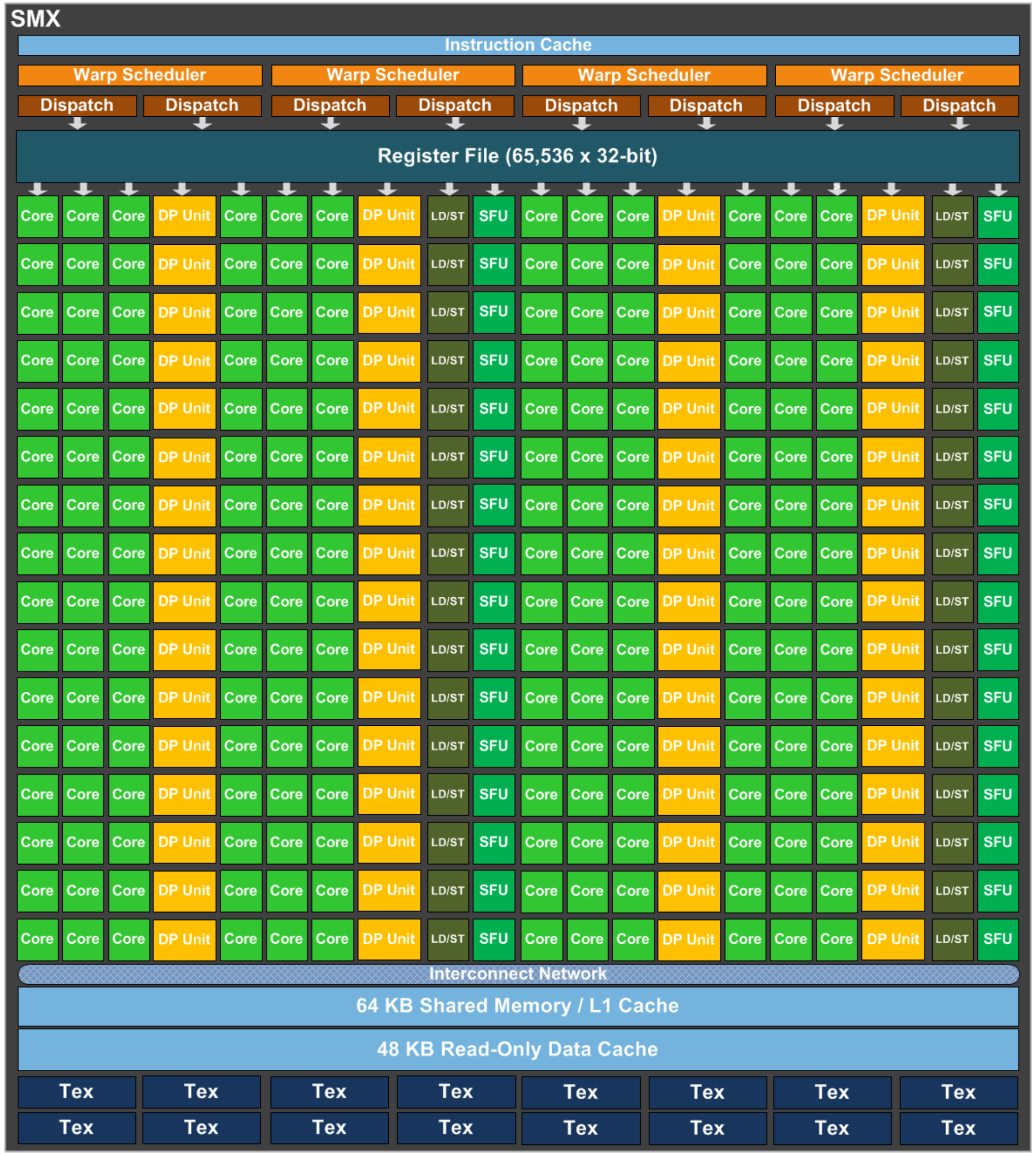
Björn Andersson
SEI CMU
Pittsburgh, USA

Fabrice Guet
Onera
Toulouse, France

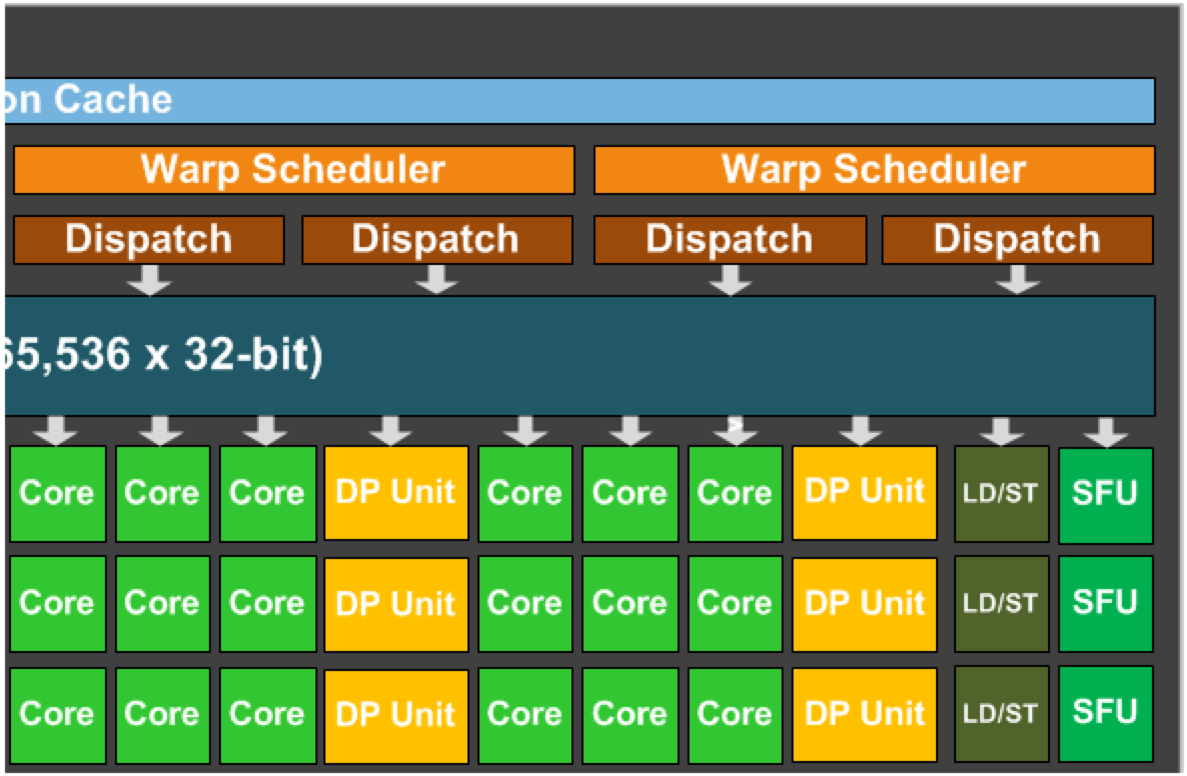
GPU architecture

© nvidia.com

GPU architecture



GPU architecture



Application areas

© nvidia.com

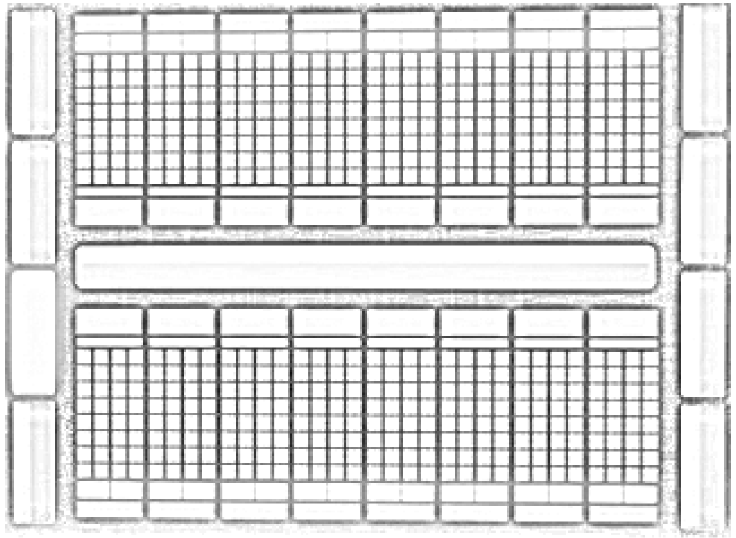
GPU architecture

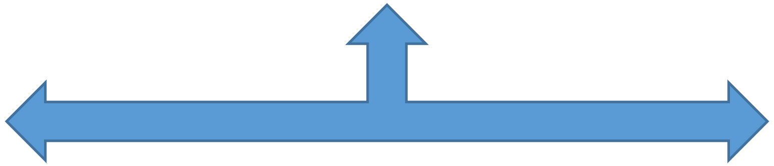
Timing Analysis

Static

Hybrid

Measurement-based





GPU Timing Analysis

Static

Makespan computation for GPU threads running on a single streaming multiprocessor
Kostiantyn Berezovskyi, Konstantinos Bletsas, Björn Andersson
Euromicro Conference on Real-time Systems (ECRTS2012)

Faster makespan estimation for GPU threads on a single streaming multiprocessor.
Kostiantyn Berezovskyi, Konstantinos Bletsas, Stefan M. Petters
18th IEEE International Conference on Emerging Technologies and Factory Automation (ETFA2013)

On Static Timing Analysis of GPU Kernels.
Vesa Hirvisalo 14th International Workshop on Worst-Case Execution Time Analysis, 2014

Hybrid

Estimating the WCET of GPU-accelerated applications using hybrid analysis.
A. Betts and A. F. Donaldson 18th Euromicro Conference on Real-time Systems (ECRTS), 2013.

MB

WCET Measurement-based and Extreme Value Theory Characterisation of CUDA Kernels
Kostiantyn Berezovskyi, Luca Santinelli, Konstantinos Bletsas, Eduardo Tovar
22nd International Conference on Real-Time Networks and Systems (RTNS 2014)

Measurement-based Probabilistic Timing Analysis for Graphics Processing Units
Kostiantyn Berezovskyi, Fabrice Guet, Luca Santinelli, Konstantinos Bletsas, Eduardo Tovar
Architecture of Computing Systems (ARCS 2016)

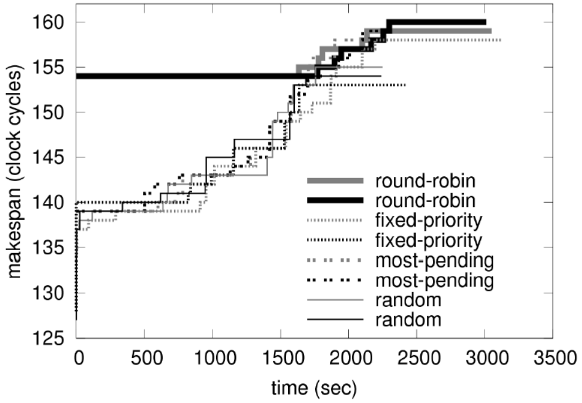
GPU Timing Analysis

Static

Optimization-based approach.

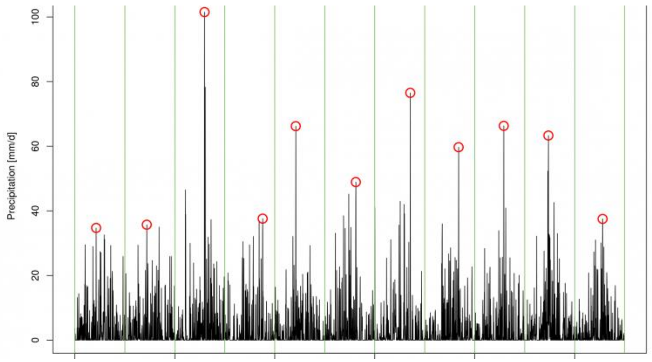
maximize $\sum_{t=1}^{T}(t \times (LS_{W,I,t} + CC_{W,I,t}))$ subject to		
iterated variables	expression for constraint	number of constraints
$\forall t$	$\sum_{w=1}^{W}\sum_{i=1}^{I} LS_{w,i,t} \leq \sigma_L$	T
$\forall t$	$\sum_{w=1}^{W}\sum_{i=1}^{I} CC_{w,i,t} \leq \sigma_C$	T
$\forall w = 1..(W-1)$	$\sum_{t=1}^{T}(t \times (LS_{w,I,t} + CC_{w,I,t})) \leq \sum_{t=1}^{T}(t \times (LS_{W,I,t} + CC_{W,I,t}))$	$W - 1$
$\forall w, t$	$\sum_{i=1}^{I} LS_{w,i,t} \leq 1$	$W \times T$
$\forall w, t$	$\sum_{i=1}^{I} CC_{w,i,t} \leq 1$	$W \times T$
$\forall w, i$	$\sum_{t=1}^{T} LS_{w,i,t} = IL_i$	$W \times I$
$\forall w, i$	$\sum_{t=1}^{T} CC_{w,i,t} = IC_i$	$W \times I$
$\forall w, i = 1..(I-1)$	$\sum_{t=1}^{T}(t \times (LS_{w,i,t} + CC_{w,i,t})) < \sum_{t=1}^{T}(t \times (LS_{w,i+1,t} + CC_{w,i+1,t}))$	$W \times (I-1)$
$\forall t$	$E_t \geq 1 - \sigma_L + \sum_{w'=1}^{W}\sum_{i=1}^{I} LS_{w',i,t}$	T
$\forall t$	$E_t \times \sigma_L \leq \sum_{w'=1}^{W}\sum_{i=1}^{I} LS_{w',i,t}$	T
$\forall w, i, t$	$\frac{1}{2}(TL_{w,i,t} + E_t) \leq TL_{w,i,t} \vee E_t \leq TL_{w,i,t} + E_t$	$W \times I \times T$
$\forall t$	$G_t \geq 1 - \sigma_C + \sum_{w'=1}^{W}\sum_{i=1}^{I} CC_{w',i,t}$	T
$\forall t$	$G_t \times \sigma_C \leq \sum_{w'=1}^{W}\sum_{i=1}^{I} CC_{w',i,t}$	T
$\forall w, i, t$	$\frac{1}{2}(TC_{w,i,t} + G_t) \leq TC_{w,i,t} \vee G_t \leq TC_{w,i,t} + G_t$	$W \times I \times T$

Metaheuristics-based approach.

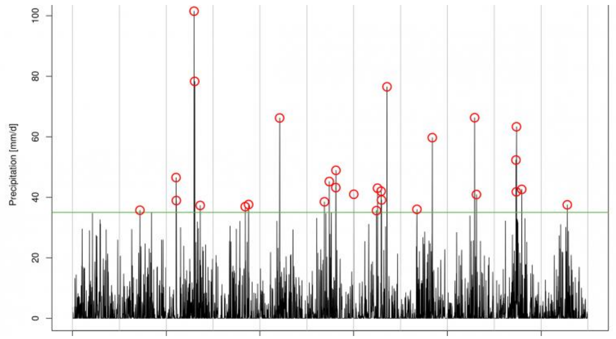


Measurement-based

Block Maxima approach.



Peaks-Over-Threshold approach.



Tandem of static approaches

Optimization-based approach.

+

Metaheuristics-based approach.

=

Upper-bound

on the unknown WCET

Lower-bound

Optimization-based approach

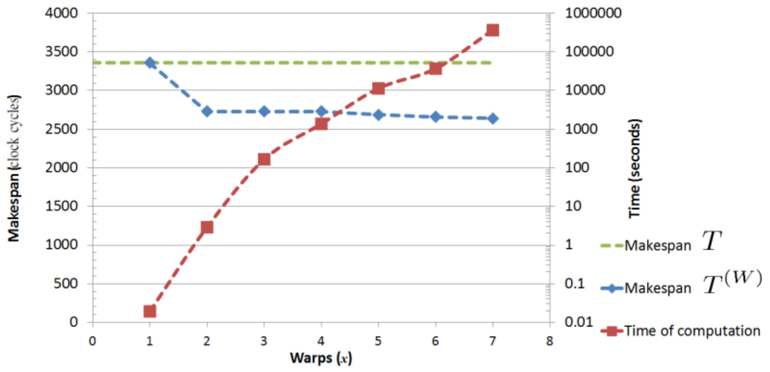
Makespan computation for GPU threads running on a single streaming multiprocessor

Kostiantyn Berezovskyi, Konstantinos Bletsas, Björn Andersson

Euromicro Conference on Real-time Systems (ECRTS2012)

maximize $\sum_{t=1}^{T}(t \times (LS_{W,I,t} + CC_{W,I,t}))$ subject to

iterated variables	expression for constraint	number of constraints
$\forall t$	$\sum_{w=1}^{W}\sum_{i=1}^{I} LS_{w,i,t} \leq \sigma_L$	T
$\forall t$	$\sum_{w=1}^{W}\sum_{i=1}^{I} CC_{w,i,t} \leq \sigma_C$	T
$\forall w = 1..(W-1)$	$\sum_{t=1}^{T}(t \times (LS_{w,I,t} + CC_{w,I,t})) \leq \sum_{t=1}^{T}(t \times (LS_{W,I,t} + CC_{W,I,t}))$	$W-1$
$\forall w, t$	$\sum_{i=1}^{I} LS_{w,i,t} \leq 1$	$W \times T$
$\forall w, t$	$\sum_{i=1}^{I} CC_{w,i,t} \leq 1$	$W \times T$
$\forall w, i$	$\sum_{t=1}^{T} LS_{w,i,t} = IL_i$	$W \times I$
$\forall w, i$	$\sum_{t=1}^{T} CC_{w,i,t} = IC_i$	$W \times I$
$\forall w, i = 1..(I-1)$	$\sum_{t=1}^{T}(t \times (LS_{w,i,t} + CC_{w,i,t})) < \sum_{t=1}^{T}(t \times (LS_{w,i+1,t} + CC_{w,i+1,t}))$	$W \times (I-1)$
$\forall t$	$E_t \geq 1 - \sigma_L + \sum_{w'=1}^{W}\sum_{i=1}^{I} LS_{w',i,t}$	T
$\forall t$	$E_t \times \sigma_L \leq \sum_{w'=1}^{W}\sum_{i=1}^{I} LS_{w',i,t}$	T
$\forall w, i, t$	$\frac{1}{2}(TL_{w,i,t} + E_t) \leq TL_{w,i,t} \vee E_t \leq TL_{w,i,t} + E_t$	$W \times I \times T$
$\forall t$	$G_t \geq 1 - \sigma_C + \sum_{w'=1}^{W}\sum_{i=1}^{I} CC_{w',i,t}$	T
$\forall t$	$G_t \times \sigma_C \leq \sum_{w'=1}^{W}\sum_{i=1}^{I} CC_{w',i,t}$	T
$\forall w, i, t$	$\frac{1}{2}(TC_{w,i,t} + G_t) \leq TC_{w,i,t} \vee G_t \leq TC_{w,i,t} + G_t$	$W \times I \times T$



Exact

Upper bound on the

worst-case makespan

Optimization problem

maximize $\sum_{t=1}^{T}(t \times (LS_{W,I,t} + CC_{W,I,t}))$ subject to		
iterated variables	expression for constraint	number of constraints
$\forall t$	$\sum_{w=1}^{W}\sum_{i=1}^{I} LS_{w,i,t} \leq \sigma_L$	T
$\forall t$	$\sum_{w=1}^{W}\sum_{i=1}^{I} CC_{w,i,t} \leq \sigma_C$	T
$\forall w = 1..(W-1)$	$\sum_{t=1}^{T}(t \times (LS_{w,I,t} + CC_{w,I,t})) \leq \sum_{t=1}^{T}(t \times (LS_{W,I,t} + CC_{W,I,t}))$	$W-1$
$\forall w, t$	$\sum_{i=1}^{I} LS_{w,i,t} \leq 1$	$W \times T$
$\forall w, t$	$\sum_{i=1}^{I} CC_{w,i,t} \leq 1$	$W \times T$
$\forall w, i$	$\sum_{t=1}^{T} LS_{w,i,t} = IL_i$	$W \times I$
$\forall w, i$	$\sum_{t=1}^{T} CC_{w,i,t} = IC_i$	$W \times I$
$\forall w, i = 1..(I-1)$	$\sum_{t=1}^{T}(t \times (LS_{w,i,t} + CC_{w,i,t})) < \sum_{t=1}^{T}(t \times (LS_{w,i+1,t} + CC_{w,i+1,t}))$	$W \times (I-1)$
$\forall t$	$E_t \geq 1 - \sigma_L + \sum_{w'=1}^{W}\sum_{i=1}^{I} LS_{w',i,t}$	T
$\forall t$	$E_t \times \sigma_L \leq \sum_{w'=1}^{W}\sum_{i=1}^{I} LS_{w',i,t}$	T
$\forall w, i, t$	$\frac{1}{2}(TL_{w,i,t} + E_t) \leq TL_{w,i,t} \vee E_t \leq TL_{w,i,t} + E_t$	$W \times I \times T$
$\forall t$	$G_t \geq 1 - \sigma_C + \sum_{w'=1}^{W}\sum_{i=1}^{I} CC_{w',i,t}$	T
$\forall t$	$G_t \times \sigma_C \leq \sum_{w'=1}^{W}\sum_{i=1}^{I} CC_{w',i,t}$	T
$\forall w, i, t$	$\frac{1}{2}(TC_{w,i,t} + G_t) \leq TC_{w,i,t} \vee G_t \leq TC_{w,i,t} + G_t$	$W \times I \times T$

Scheduling Policy

C L L C L L   2 warps

Clock Cycle	1	2	3	4	5	6	7	8	9	10
Warp 1	C	L			L	C	L		L	
Warp 2			C	L		L	C	L		L

Non-work-conserving schedule

Clock Cycle	1	2	3	4	5	6	7	8	9
Warp 1	C	L		L	C	L		L	
Warp 2		C	L		L	C	L		L

Work-conserving schedule

$$\forall w, t \quad (TL_{w,1,t} \wedge TL_{w,2,t} \wedge \cdots \wedge TL_{w,I,t}) \vee E_t$$

INPUT		OUTPUT
A	B	$A \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

INPUT		OUTPUT
A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

Theorem 1 $\forall i \quad 1 \leq i \leq I$ *such that* $i, I \in \mathbb{N}$; $I \geq 2$ *and* $x_i, X \in \{0, 1\}$:

An inequality (23)

$$\frac{1}{I}\sum_{i=1}^{I} x_i \leq X \leq \sum_{i=1}^{I} x_i$$

is equivalent to the equality $X = \vee_{i=1}^{I} x_i$

INPUT		OUTPUT
A	B	$A \lor B$
T	T	T
T	F	T
F	T	T
F	F	F

Theorem 1 $\forall i \quad 1 \leq i \leq I$ *such that* $i, I \in \mathbb{N}$; $I \geq 2$ *and* $x_i, X \in \{0, 1\}$:
An inequality (23)

$$\frac{1}{I}\sum_{i=1}^{I} x_i \leq X \leq \sum_{i=1}^{I} x_i$$

is equivalent to the equality $X = \vee_{i=1}^{I} x_i$

Case 1: $\forall i \quad 1 \leq i \leq I \quad i \in \mathbb{N} \quad x_i = 0$

Case 2: $\exists j \quad 1 \leq j \leq I \quad j \in \mathbb{N} \quad x_j = 1$

$$(TL_{w,1,t} \wedge TL_{w,2,t} \wedge \cdots \wedge TL_{w,I,t}) \vee E_t$$

INPUT		OUTPUT
A	B	$A \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

INPUT		OUTPUT
A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

Theorem 2 $\forall i \quad 1 \leq i \leq I$ *such that* $i, I \in \mathbb{N}$; $I \geq 2$ *and* $x_i, X \in \{0,1\}$

The inequality (28)

$$-\frac{I-1}{I} + \frac{1}{I}\sum_{i=1}^{I} x_i \leq X \leq \frac{1}{I}\sum_{i=1}^{I} x_i$$

is equivalent to the equality

$$X = \wedge_{i=1}^{I} x_i$$

INPUT		OUTPUT
A	B	$A \land B$
T	T	T
T	F	F
F	T	F
F	F	F

Theorem 2 $\forall i \quad 1 \leq i \leq I$ *such that* $i, I \in \mathbb{N}$; $I \geq 2$ *and* $x_i, X \in \{0,1\}$

The inequality (28)

$$-\frac{I-1}{I} + \frac{1}{I}\sum_{i=1}^{I} x_i \leq X \leq \frac{1}{I}\sum_{i=1}^{I} x_i$$

is equivalent to the equality

$$X = \wedge_{i=1}^{I} x_i$$

Case 1: $\forall i \quad 1 \leq i \leq I \quad i \in \mathbb{N} \quad x_i = 1$

Case 2: $\exists j \quad 1 \leq j \leq I \quad j \in \mathbb{N} \quad x_j = 0$

$$(TL_{w,1,t} \wedge TL_{w,2,t} \wedge \cdots \wedge TL_{w,I,t}) \vee E_t$$

$$\forall w, t \quad \frac{1}{2}((-\frac{I-1}{I} + \frac{1}{I}\sum_{i=1}^{I} TL_{w,i,t}) + E_t) \leq$$

$$(TL_{w,1,t} \wedge TL_{w,2,t} \wedge \cdots \wedge TL_{w,I,t}) \vee E_t \leq$$

$$\frac{1}{I}\sum_{i=1}^{I} TL_{w,i,t} + E_t$$

$$Z \in \{0, 1\}:$$

$$\forall w, t \quad \frac{1}{2}((-\frac{I-1}{I} + \frac{1}{I}\sum_{i=1}^{I} TL_{w,i,t}) + E_t) \leq$$

$$Z \leq$$

$$\frac{1}{I}\sum_{i=1}^{I} TL_{w,i,t} + E_t$$

$$Z$$

$$\neq$$

$$(TL_{w,1,t} \wedge TL_{w,2,t} \wedge \cdots \wedge TL_{w,I,t}) \vee E_t$$

Theorem 3 $\forall i \quad 1 \leq i \leq I$ *such that* $i, I \in \mathbb{N}$*;* $I \geq 2$ *and* $x_i, y, Z \in \{0, 1\}$*:*
The inequality (33)

$$\frac{1}{2} \times (-\frac{I-1}{I} + \frac{1}{I} \times \sum_{i=1}^{I} x_i + y) - \frac{1}{2 \times I} < Z \leq \frac{1}{I} \times \sum_{i=1}^{I} x_i + y$$

is equivalent to the equality (34)

$$Z = (\wedge_{i=1}^{I} x_i) \vee y$$

$$L = \frac{1}{2} \times (-\frac{I-1}{I} + \frac{1}{I} \times \sum_{i=1}^{I} x_i + y) - \frac{1}{2 \times I}$$

$$R = \frac{1}{I} \times \sum_{i=1}^{I} x_i + y$$

Theorem 3 $\forall i \quad 1 \leq i \leq I$ *such that* $i, I \in \mathbb{N}$; $I \geq 2$ *and* $x_i, y, Z \in \{0, 1\}$:

The inequality (33)

$$\frac{1}{2} \times (-\frac{I-1}{I} + \frac{1}{I} \times \sum_{i=1}^{I} x_i + y) - \frac{1}{2 \times I} < Z \leq \frac{1}{I} \times \sum_{i=1}^{I} x_i + y$$

is equivalent to the equality (34)

$$Z = (\wedge_{i=1}^{I} x_i) \vee y$$

$$(TL_{w,1,t} \wedge TL_{w,2,t} \wedge \cdots \wedge TL_{w,I,t}) \vee E_t$$

$$(TL_{w,1,t} \vee E_t) \wedge (TL_{w,2,t} \vee E_t) \wedge \cdots \wedge (TL_{w,I,t} \vee E_t)$$

$$(TL_{w,1,t} \vee E_t) \wedge (TL_{w,2,t} \vee E_t) \wedge \cdots \wedge (TL_{w,I,t} \vee E_t)$$

$$\forall w, i, t \quad \frac{1}{2}(TL_{w,i,t} + E_t) \leq TL_{w,i,t} \vee E_t \leq TL_{w,i,t} + E_t$$

$$(TL_{w,1,t} \wedge TL_{w,2,t} \wedge \cdots \wedge TL_{w,I,t}) \vee E_t$$

VS

$$(TL_{w,1,t} \vee E_t) \wedge (TL_{w,2,t} \vee E_t) \wedge \cdots \wedge (TL_{w,I,t} \vee E_t)$$

Thank You!

Questions?