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Abstract

Network diversity multiple access (NDMA) is the family of algorithms with the highest potential throughput in the literature of signal-processing assisted random access protocols. NDMA uses the concept of protocol-induced retransmissions to create an adaptive source of diversity. This diversity is used to resolve packet collisions employing signal separation tools without the explicit need (or as a complement) of a multiple antenna receiver. This paper proposes a further improvement on the performance of NDMA by allowing each terminal access to an outdated copy of its individual channel state information (CSI). Based on this decentralized CSI, each terminal conveniently decides to transmit only if the estimated channel gain surpasses a threshold that is optimized to maximize performance. This ensures that the probability of terminal presence detection, and thus the probability of correct estimation of the collision multiplicity are considerably improved at the receiver end. The paper is focused on the modelling of the receiver operational characteristic (ROC) of the terminal presence detector considering that the CSI used by each terminal is potentially inaccurate (outdated) due to feedback delay. The results indicate that when the correlation coefficient that describes the accuracy of the available CSI tends to zero, the scheme degrades into the conventional NDMA. By contrast, when the quality of the channel state information improves, the throughput can nearly achieve the nominal channel rate (minimum throughput penalty). The selection of the detector thresholds for channel gain and terminal presence is optimized to maximize system performance.

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***Abstract*—Network diversity multiple access (NDMA) is the family of algorithms with the highest potential throughput in the literature of signal-processing assisted random access protocols. NDMA uses the concept of protocol-induced retransmissions to create an adaptive source of diversity. This diversity is used to resolve packet collisions employing signal separation tools without the explicit need (or as a complement) of a multiple antenna receiver. This paper proposes a further improvement on the performance of NDMA by allowing each terminal access to an outdated copy of its individual channel state information (CSI). Based on this decentralized CSI, each terminal conveniently decides to transmit only if the estimated channel gain surpasses a threshold that is optimized to maximize performance. This ensures that the probability of terminal presence detection, and thus the probability of correct estimation of the collision multiplicity are considerably improved at the receiver end. The paper is focused on the modelling of the receiver operational characteristic (ROC) of the terminal presence detector considering that the CSI used by each terminal is potentially inaccurate (outdated) due to feedback delay. The results indicate that when the correlation coefficient that describes the accuracy of the available CSI tends to zero ($\rho \rightarrow 0$), the scheme degrades into the conventional NDMA. By contrast, when the quality of the channel state information improves ($\rho \rightarrow 1$), the throughput can nearly achieve the nominal channel rate (minimum throughput penalty). The selection of the detector thresholds for channel gain and terminal presence is optimized to maximize system performance.**

I. Introduction

A. Network diversity multiple access (NDMA)

The concept of multi-packet reception (MPR) is one of the main research lines in the evolution of random access protocols. MPR represents the ability of the physical (PHY) layer to correctly decode concurrent transmissions that otherwise were conventionally discarded (i.e., unresolved collisions). MPR can be achieved via power capture or mainly by multiple antenna reception systems (also known as MIMO or multiple-input multiple-output systems in centralized networks). The consequences of MPR in random access are expected to be large, since collisions are the main source of inefficiency in this type of systems. Increasing capacity would place random access as a competing technology for different services in 5G and future networks, where signalling load will be limited due to the large number terminals that are expected to be connected to the cloud. Random access with MPR will therefore receive renewed attention in the years to come.

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Cross-layer design is an important tool in the study of random access networks enabled with MPR [1]. Correct reception in this type of system depends on two aspects: 1) *PHY-layer performance*, and 2) *traffic load at the medium access control (MAC) layer*. A relevant breakthrough in the literature of cross-layer random access was the work in [2], where collisions were resolved by means of a new type of diversity based on retransmissions. The algorithm was called network diversity multiple access (NDMA). In NDMA, terminal retransmissions are used to create a virtual MIMO system from which colliding signals can be recovered via source separation. Signals with collisions that can not be resolved within the same time-slot are thus not discarded as in conventional ALOHA-type protocols. Instead, they are stored in memory for further processing. Initially, they are used to estimate the collision multiplicity. Based on this extracted information, the base station (BS) proceeds to request further retransmissions from the contending terminals until all the collected signals mimic a *full-rank* MIMO system. The BS then uses all the stored signals to recover the colliding transmissions via source separation. A cooperative NDMA protocol coined ALLIANCES was later proposed in [3]. An NDMA protocol assisted by multi-packet reception was presented in [4] with a finite user population model, and in [5] with an infinite user population model. Stability features of NDMA with perfect collision multiplicity estimation and perfect packet reception can be found in [6]. A Markov model for the study of the stability of NDMA with imperfect collision multiplicity estimation was presented in [7]. NDMA incorporating MPR and successive interference cancellation was proposed in [8] achieving throughput values potentially higher than the nominal rate of the channel.

B. Paper contributions and organisation

This paper presents the analysis of the training-based version of the NDMA protocol in non-dispersive Rayleigh fading channels under the assumption that terminals have access to an imperfect (outdated) version of their own channel state information (CSI). Based on this decentralized CSI, terminals decide to transmit when the estimated channel strength surpasses a given threshold that maximizes the probability of correct terminal presence detection at the receiver side. An important part of this work is devoted to the derivation of the receiver operating characteristic (ROC) of the terminal presence detector and on the derivation of the statistics of the received detection signal conditional on the decisions based on imperfect . A correlation linear model is used to introduce in the analysis the accuracy of the estimated CSI, thus mimicking a feedback channel affected by delay.

This paper is organized as follows. Section II presents the system model. Section III and IV presents the modelling of the signal detection and MPR models, respectively. Section V presents the analysis of the terminal presence detector considering imperfect CSI. Section VI presents the metrics that will be used to evaluate system performance. Section VII presents a discussion of results using different network settings. Finally, Section VIII presents the conclusions.

II. System model

A. System scenario and channel model

Consider the slotted wireless random access network depicted in Fig. 1 with a set of J buffered one-antenna terminals and one central node or base station (BS) with one receiver antenna. The channel between terminal j and the BS is denoted by h_j. All channel envelopes are assumed to be block-fading and non-dispersive with Rayleigh statistics and variance γ: $h_j \sim \mathcal{CN}(0, \gamma)$[1]. It is assumed that each terminal has access to a copy (outdated) of its own CSI. This estimated channel variable will be denoted by $\hat{h}_j$, and will be also modelled as a circular complex Gaussian process with variance γ: $\hat{h}_j \sim \mathcal{CN}(0, \gamma)$. To model the inaccuracy of the CSIT (channel state information at the transmitter side) a linear correlation model will be employed:

$$h_j = \rho \hat{h}_j + \sqrt{1-\rho^2}\chi_j, \tag{1}$$

where the variable χ_j is also a circular complex Gaussian random variable with zero mean and variance γ. Terminals use this estimated CSIT to decide when it is more convenient to transmit a packet towards the BS.

B. Protocol overview

All terminals will be assumed to experience a Poisson-distributed[2] packet random arrival process described by the parameter λ. Since NDMA exploits the time domain to create diversity, the number of time-slots used to resolve any collision will be described by a random variable denoted here by l. The period of time used to resolve a packet collision will be called contention resolution period or *epoch-slot* (see Fig. 1). At the beginning of every epoch-slot each terminal will attempt a packet transmission when the following two conditions are met: 1)when the CSIT is above the pre-set threshold ($\hat{z}_j \geq \hat{\beta}$), and 2) when a packet is available in the queue for transmission. Therefore, the total transmission probability can be written as $p_t = p\Pr\{\hat{z}_j \geq \hat{\beta}\}$, where p is the probability of packet being available at the queue for transmission. In the steady-state and under Poisson arrival distribution it has been proved that the following traffic balance equation holds [6]:

$$p = \lambda E[l], \tag{2}$$

where $E[\cdot]$ is the statistical average operator. For convenience let us denote the set of contending terminals during the first time-slot of any epoch-slot as $\mathcal{T}$. At the beginning of every epoch-slot, the BS proceeds to estimate the collision multiplicity as described in detail in Section V. The BS obtains an estimation $\hat{K} = |\hat{\mathcal{T}}|$ of the collision multiplicity $K = |\mathcal{T}|$, where $\hat{\mathcal{T}}$ indicates the set of terminals detected as active and $|\cdot|$ is the set cardinality operator when applied to a set variable. Once this information has been obtained, the BS proceeds to calculate the number of retransmissions required to resolve the collision. Since the BS has one receive antenna, the number of transmissions (including the initial transmission plus retransmissions) required in the non-blind version of NDMA is given by $\hat{K}$ [7]. This means that the number of diversity sources must be greater than or equal to the estimated collision multiplicity.

In NDMA, having more diversity sources than contending signals is necessary to maximize the probability that the channel matrix is full-rank, which in turn improves the probability of success of the source separation stage [2]. To request a retransmission for diversity purposes, the BS simply indicates with an ideal and instantaneous binary feedback flag $\xi \in \{0, 1\}$ at the end of each time-slot to all the contending terminals that retransmission is required in the next time slot. The feedback flag is kept "*on*" ($\xi = 1$) until all necessary retransmissions have been collected. No other terminals are allowed to transmit in the current epoch-slot. These protocol steps are repeated for subsequent epoch-slots.

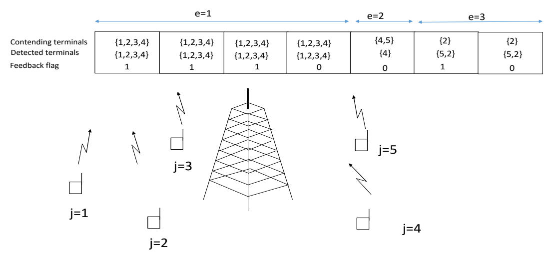

Fig. 1. Random access network assisted by retransmission diversity.

To further illustrate the mechanism of the protocol, Fig. 1 displays three realizations of epochs. In the first epoch-slot ($e = 1$), four terminals $\mathcal{T} = \{1, 2, 3, 4\}$ have collided in the first time-slot. The figure indicates the following variables of the system: the set of contending terminals $\mathcal{T}$, the set of terminals detected as active $\hat{\mathcal{T}}$, and the binary feedback flag ξ used to request retransmissions. Since four signals need to be recovered in the first epoch, then three more retransmissions are needed to potentially resolve the collision. Note that in this first epoch the set of detected terminals is identical to the set of contending terminals ($\hat{\mathcal{T}} = \mathcal{T}$), which means no detection

[1]Rayleigh assumption allows for closed-form expressions that help in visualize the main merits of the protocol.

[2]Poisson distributed traffic facilitates steady-state analysis of the protocol as shown in [2].

errors occurred. In this case, the number of collected signals is equal to four, which is enough to attempt the recovery of the four contending signals. Also note that the binary feedback is only set to $\xi = 1$ at the end of the first three time-slots. Once the third and last retransmission has been received, the value is set to $\xi = 0$, which means that the current epoch has finalized so the contending terminals stop retransmitting information while the other terminals are allowed to transmit in a new epoch-slot. Epochs $e = 2$ and $e = 3$ show, respectively, the cases where one contending terminal was not correctly detected as active and when a non-contending terminal is incorrectly detected as active (false alarm), thus leading to the loss of all contending packets.

III. Signal model for terminal presence detection

Each terminal is pre-assigned with an orthogonal code with J symbols: $\mathbf{w}_j = [w_j(0), \ldots, w_j(J-1)]^T$, where $(\cdot)^T$ is the vector transpose operator. This code is attached as header of each packet transmission, and is employed for purposes of presence detection (collision multiplicity estimation) and channel estimation. The orthogonality condition of the set of codes is given by: $\mathbf{w}_j^H \mathbf{w}_k = \begin{cases} J, & k = j \\ 0, & k \neq j \end{cases}$, where $(\cdot)^H$ is the Hermitian vector transpose operator. The received signal coming from all the headers of the set of colliding terminals (denoted here by $\mathcal{T}$) can be written as:

$$\mathbf{y}^{(h)} = \sum_{j \in \mathcal{T}} h_j \mathbf{w}_j + \mathbf{v}^{(h)}, \tag{3}$$

where $\mathbf{v}^{(h)} = [v^{(h)}(0), \ldots, v^{(h)}(J-1)]^T$ is the zero-mean and white complex Gaussian noise vector in the header with variance σ_v^2. This means that $\mathbf{v}^{(h)} \sim \mathcal{CN}(\mathbf{0}_J, \sigma_v^2 \mathbf{I}_J)$, where $\mathbf{0}_J$ and $\mathbf{I}_J$ are, respectively, the vector of J zeros and the identity matrix of size J. The BS uses a matched-filter operation ($\mathbf{w}_j^T \mathbf{y}^{(h)}$) to extract the presence information of each terminal j. The result is computed as follows:

$$z_j = |\mathbf{w}_j^T \mathbf{y}^{(h)}|^2. \tag{4}$$

The presence detection variable z_j for terminal j in (4) is compared to a detection threshold β to decide whether terminal j is present or not in the collision. If $z_j < \beta$, then terminal j is considered as inactive or not present in the collision: $j \notin \widehat{\mathcal{T}}$, where $\widehat{\mathcal{T}}$ is the estimated set of contending terminals. Otherwise, if $z_j > \beta$, then terminal j is considered as active or present in the collision ($j \in \widehat{\mathcal{T}}$). The estimated set of contending terminals can be therefore defined as the set $\hat{\mathcal{T}}$ of all the terminals whose detection variable z_j exceeds the detection threshold:

$$\hat{\mathcal{T}} = \{j | z_j \geq \beta\}. \tag{5}$$

Since this detection process is prone to errors due to channel fading and noise, two conditional terminal presence detection cases can be identified: 1) terminal j can be correctly detected as active with probability P_D provided the terminal has transmitted a packet, and 2) terminal j is incorrectly detected as active with probability P_F (probability of false alarm) provided the terminal did not transmit a packet. By detecting the presence of each one of the contending terminals, the BS also has an estimation $\hat{K} = |\widehat{\mathcal{T}}|$ of the collision multiplicity $K = |\mathcal{T}|$, where $|\cdot|$ is the set cardinality operator, $\widehat{K}$ is the estimated number of contending signals, and K is the number of contending signals.

IV. Signal model for multi-packet reception

Each terminal j transmits packets with Q QAM symbols denoted by $\mathbf{x}_j = [x_j(0), x_j(1) \ldots x_j(Q-1)]^T$. Considering unitary packet power transmission $E[\mathbf{x}_j^H \mathbf{x}_j] = 1$, the signal vector received at the beginning of an epoch is given by:

$$\mathbf{y} = \sum_{j \in \mathcal{T}} h_j \mathbf{x}_j + \mathbf{v}, \tag{6}$$

where $\mathbf{v} = [v(0), v(1) \ldots, v(Q-1)]^T$ is the zero-mean and white complex Gaussian noise vector: $\mathbf{v} \sim \mathcal{CN}(\mathbf{0}_Q, \sigma_v^2 \mathbf{I}_Q)$. The BS proceeds to estimate the collision multiplicity by means of terminal activity detection (explained in the previous section) and requests the number of necessary retransmission (given by $\widehat{K} - 1$) to resolve the collision. All the collected (re)transmissions are kept in memory to create a virtual MIMO system that can be expressed as follows [2] [6]: $\mathbf{Y}_{\widehat{K} \times Q} = \mathbf{H}_{\widehat{K}K} \mathbf{S}_{K \times Q} + \mathbf{V}_{\widehat{K} \times Q}$, where $\mathbf{Y}$ is the array formed by the collection of all received signals from all the $\widehat{K}$ time-slots of the epoch, $\mathbf{H}$ is the mixing matrix or MIMO (multiple-input multiple-output) channel, $\mathbf{S}$ is the array of stacked packets from all the contending terminals, each one with Q symbols, and finally $\mathbf{V}$ is the collected Gaussian noise components. The mixing matrix $\mathbf{H}$ can be estimated by using the outcome of the matched filter operation from each antenna and from each collected retransmission. The estimate $\hat{\mathbf{H}}$ can be used to recover the contending packets. The contending signals can be estimated at the BS by means of a linear decoding matrix $\mathbf{A}$. The decoding matrix can be obtained using zero-forcing, minimum mean square error (MMSE) or other criterion. Conventionally, in NDMA the packet reception performance is approximated by the terminal presence detection process [2]. If all the colliding terminals are correctly detected as active and all the non-colliding terminals are correctly detected as idle, then all the signals involved in the collision are considered to be correctly received by the BS. Otherwise it is assumed that all signals are lost. This approximation has been proved valid at high values of SNR [2].

V. Detector performance model

NDMA highly depends on the performance of the terminal presence detector. Any detection error, usually leads either to the loss of a significant percentage of the colliding signals in case of underestimation of collision multiplicity, or to the waste of transmission resources in case of overestimation of multiplicity. This section deals with the statistical modelling of the terminal presence detector in (4). This will be useful for subsequent calculations and design of the MAC layer. The probability of false alarm (P_F) of a terminal that did not transmit a signal while still being detected as active can be defined more formally as follows:

$$P_F = \Pr\{z_j > \beta | j \notin \mathcal{T}\} = \Pr\{j \in \hat{\mathcal{T}}_d | j \notin \mathcal{T}\}, \quad (7)$$

which is the probability that the detection variable z_j exceeds the detection threshold β, conditional on terminal j not being one of the contending terminals. Since noise is Gaussian distributed, the detection variable z_j in (4) follows a central chi-square distribution, and therefore the probability of false alarm can be expressed in closed-form [2]:

$$P_F = e^{-\frac{\beta}{J\sigma_v^2}}. \quad (8)$$

Similarly, the probability of detection of terminal j, conditional on terminal j being one of the contending terminals can be defined as:

$$P_D = \Pr\{z_j \geq \beta | j \in \mathcal{T}\} = \Pr\{j \in \hat{\mathcal{T}}_d | j \in \mathcal{T}\}. \quad (9)$$

To obtain an analytic expression for P_D, let us substitute the expression of the signal header model of (3) in the expression of the signal presence detection model in (4):

$$z_j = |Jh_j + \sum_{l=1}^{J} v_j^{(h)}(l)|^2$$

By substituting the channel correlation model from (1) in the previous expression leads to:

$$z_j = |J\rho\hat{h}_j + \theta_j|^2 \quad (10)$$

where $\theta_j = J\sqrt{1-\rho^2}\chi_j + \sum_{l=1}^{J} v_j^{(h)}(l)$. Using the properties of independent complex Gaussian distributions, it follows that θ_j can be modelled as an independent complex Gaussian variable with parameter $\tilde{\gamma}$: $\theta_j \sim \mathcal{CN}(0, \tilde{\gamma})$, where $\tilde{\gamma} = J^2\gamma(1-\rho^2) + J\sigma_v^2$. Consider now the previous expression conditional on an instance of the random variable $\hat{h}$. Under this assumption, the expression in (10) becomes the square of a Gaussian complex variable θ_j with mean $J\rho\hat{h}_j$. The detection variable z_j conditional on $\hat{h}_j$ has thus a non-central chi-square distribution with two degrees of freedom. The conditional characteristic function (CF) is thus given by [9]:

$$\Psi_{z_j|\hat{h}_j}(i\omega) = (1 - i\omega\tilde{\gamma})^{-1} e^{\frac{i\omega J^2\rho^2|\hat{h}_j|^2}{1-i\omega\tilde{\gamma}}}, \quad (11)$$

where $\Psi_{A|B}$ denotes the CF of random variable A conditional on random variable B, for any random variables A and B. By substituting $\hat{z}_j = J^2|\hat{h}_j|^2$ in the previous expression it leads to:

$$\Psi_{z_j|\hat{z}_j}(i\omega) = (1 - i\omega\tilde{\gamma})^{-1} e^{\frac{i\omega\rho^2\hat{z}_j}{1-i\omega\tilde{\gamma}}}. \quad (12)$$

The unconditional CF of the instantaneous detection variable z_j can be obtained by averaging the previous expression over the PDF of $\hat{z}_j$ conditional on $\hat{z}_j \geq \hat{\beta}$, which denotes the transmission of a terminal when the estimated CSIT is above the threshold $\hat{\beta}$. Therefore, the new CF can be obtained as follows:

$$\Psi_{z_j|\hat{z}_j \geq \hat{\beta}}(i\omega) = \int_{\hat{\beta}}^{\infty} (1 - i\omega\tilde{\gamma})^{-1} e^{\frac{i\omega\rho^2\hat{z}_j}{1-\tilde{\gamma}}} f_{\hat{z}_j}(\hat{z}_j) d\hat{z}_j, \quad (13)$$

which after the integration (see the Appendix) becomes:

$$\Psi_{z_j|\hat{z}_j \geq \hat{\beta}}(i\omega) = \frac{e^{-\frac{\hat{\beta}(1-i\omega\dot{\gamma})}{\check{\gamma}(1-i\omega\tilde{\gamma})}}}{(1 - i\omega\dot{\gamma})}, \quad (14)$$

where $\dot{\gamma} = \rho_c^2\check{\gamma} + \tilde{\gamma}$, and $\check{\gamma} = J^2\gamma$. The back-transform of the previous expression can be proved (see Appendix) to yield the following CCDF:

$$\bar{F}_{z_j}(y) = \bar{W}\left(\frac{-\hat{\beta}}{\check{\gamma}}, \dot{\gamma}, \tilde{\gamma}, y\right),$$

where

$$\bar{W}(a,b,c,y) = e^{-\frac{y}{b}} + \sum_{r=1}^{\infty} \frac{a^r}{r!} \sum_{l=0}^{r-1} \binom{r-1}{l} b^l$$

$$\times \sum_{k=0}^{l} \binom{l}{k} \sum_{s=0}^{r-1-l} \left(\frac{(-1)^l y^s (r+l)!}{c^{r+l} r! s!} e^{-\frac{y}{c}}\right). \quad (15)$$

See Appendix for details of the derivation of $\bar{W}(a,b,c,y)$. The probability of correct detection can be therefore expressed as $P_D = \bar{F}(\beta)$. This concludes the derivation of the ROC of the terminal presence activity detector with imperfect CSIT.

VI. Performance metrics

A. Throughput

Packet throughput is defined here as the ratio of the average number of correctly received packets per epoch-slot to the average length of an epoch-slot. Consider that $E[t_d]$ is the average number of correctly received packets per epoch and $E[l]$ is the average length (in time-slots) of an epoch, then throughput can be expressed as follows:

$$T = \frac{E[t_d]}{E[l]}. \quad (16)$$

The numerator of (22) can expressed as the average probability of correct terminal presence detection provided all other terminals also experience a correct presence detection (each one with probability P_u) whether they are correctly detected when they have an active transmission or they are not incorrectly detected (false alarm) when they were not engaged in transmission [7]:

$$E[t_d] = Jp_t P_D P_u^{J-1}, \quad (17)$$

where $p_t = p\Pr\{\hat{z}_j \geq \hat{\beta}\}$ is the total probability of terminal packet transmission, and where the total probability of correct terminal presence detection or correct contribution P_u is given by the probability of correct detection in case of transmission plus the probability of not incurring in false alarm in case of no transmission [7]:

$$\begin{aligned} P_u &= \Pr\{j \in \hat{\mathcal{T}} | j \in \mathcal{T}\}\Pr\{j \in \mathcal{T}\} \\ &\quad + \Pr\{j \notin \hat{\mathcal{T}} | j \notin \mathcal{T}\}\Pr\{j \notin \mathcal{T}\} \\ &= P_D p_t + \bar{P}_F \bar{p}_t, \end{aligned} \quad (18)$$

where $\bar{a} = 1 - a$ for any a. This means that $\bar{p}_t = 1 - p_t$ and $\bar{P}_F = 1 - P_F$. The average length of an epoch in the denominator of (16) can be obtained by averaging over all possible cases of terminal activity detection, i.e., when an active terminal is correctly detected as active, or when an inactive terminal is incorrectly detected as active (false alarm). We recall that the number of time-slots of each epoch is determined by the number of retransmissions necessary to attempt to create a full-rank MIMO system, which in our setting is given by $\widehat{K}$ [7]. The probability mass function (PMF) of length of an epoch l is thus given by:

$$\Pr\{l = m\} = \begin{cases} \Pr\{\hat{K} = m\}, & m > 1 \\ \Pr\{\hat{K} = 0\} + \Pr\{\hat{K} = 1\}, & m = 1 \end{cases} \quad (19)$$

It can be also proved that $\hat{K}$ has a binomial distribution with parameter $P_A = p_t P_D + \bar{p}_t P_F$, which can be written as $\Pr\{\hat{K} = k\} = \begin{pmatrix} J \\ k \end{pmatrix} P_A^k \bar{P}_A^{J-k}, \quad k = 0, \ldots J$. The parameter P_A is thus regarded as the total probability of terminal activity detection, and is given by the probability of correct detection in case of transmission plus the probability of false alarm in case of no transmission:

$$P_A = \Pr\{j \in \hat{\mathcal{T}}\} = \Pr\{j \in \hat{\mathcal{T}} | j \in \mathcal{T}\}\Pr\{j \in \mathcal{T}\} + \Pr\{j \in \hat{\mathcal{T}} | j \notin \mathcal{T}\}\Pr\{j \notin \mathcal{T}\} = p_t P_D + \bar{p}_t P_F. \quad (20)$$

Therefore, average length of an epoch $E[l]$ can be obtained by averaging over the PMF of l in (19), which yields:

$$E[l] = J P_A + \bar{P}_A^J, \quad (21)$$

where the second term $\bar{P}_A^J$ stands for the contribution of one time slot in case any terminal is detected as active: $\Pr\{\hat{K} = 0\} = \bar{P}_A^J$. The throughput of the system in (16) can be thus rewritten as follows:

$$T = \frac{J p_t P_D P_u^{J-1}}{J P_A + \bar{P}_A^J} = \frac{J p_t P_D (P_D p_t + \bar{P}_F \bar{p}_t)^{J-1}}{J P_A + \bar{P}_A^J}. \quad (22)$$

The effects of decentralized CSIT are mainly absorbed in the transmission probability p_t and the probability of detection in case of transmission P_D.

VII. Results

This section presents some simulation results that confirm the potential advantages of using CSIT to improve the detection probability of the NDMA protocol. Let us consider a network setting with $J = 16$ users experiencing an average SNR ($\frac{\gamma}{\sigma_v^2}$) of 0 dB. The results were obtained for different values of the correlation coefficient (ρ) that describes the accuracy of the CSIT used by terminals. The results are displayed in Fig. 2 showing the achieved packet throughput versus different values of traffic load ($J\lambda$). The results for the conventional NDMA protocol are also shown in the figure being displayed with a tag $\rho = 0$. The results clearly show the advantage of using CSIT for terminals to decided when it is more convenient to transmit. The throughput gains increase as the value of ρ increases. Note that at low values of traffic load all systems converge to the conventional NDMA solution. The gains increase mainly at high values of traffic load. The range of values of traffic load that show gains in performance decrease with lower values of ρ. For values of ρ near to 0.7 the gains are limited to very high traffic loads. It is also important to notice that all schemes using the control transmission proposed in this paper show performance below the maximum channel rate of one packet per time slot. By contrast the conventional NDMA extends its performance beyond this value. This contradictory behaviour can be explained as follows: in NDMA the finite SNR regime causes some packets not being correctly detected at the BS. This means that even in full traffic load, the system does not detect all the transmitted packets, thus leading the system to believe that there is indeed lower traffic being transmitted. This allows conventional NDMA to show a performance beyond the value of one packet per time slot. In our proposed system, terminals are only allowed to transmit when their copy of the channel state information is above a threshold that maximizes probability of detection at the BS. Therefore, the transmission control system will automatically limit the performance for values of traffic load always below one packet per time slot. The results presented in Fig. 2 consider the simultaneous optimization of the two thresholds used in the theoretical analysis: $\hat{\beta}$ which is the threshold used at the terminal side to decide when channel conditions are good enough to enable transmission, and β which is the energy detection threshold at the BS side to decide when a terminal is present in a collision. The gains of the proposed approach have also been found to decline considerable for larger values of SNR and for values of $\rho < 0.5$.

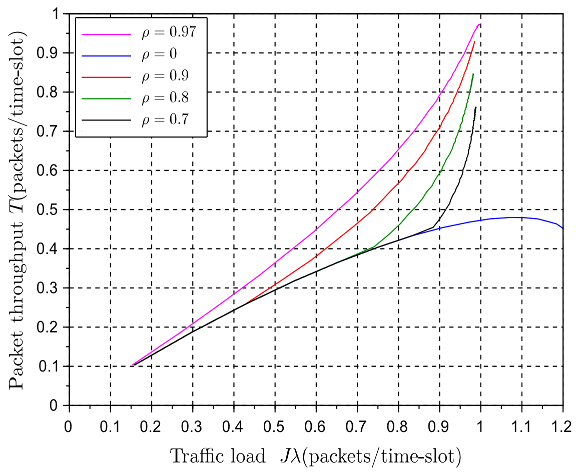

Fig. 2. Packet throughput (T) versus traffic load ($J\lambda$) for various values of the correlation coefficient between the instantaneous and the estimated CSI.

VIII. Conclusions

This paper presented an extension of the analysis of the family of random access protocols based on retransmission diversity also know as NDMA (network diversity multiple access protocols). This extension considers that terminals have access to an outdated copy of their own channel state information. If the instantaneous channel power indicator of a given terminal is above a given threshold the it is allowed to transmit. This threshold is optimized to maximize the probability of correct presence detection at the BS. This yields an important improvement on the throughput performance of NDMA. All protocol analysis is modified to consider an imperfect version (using a correlation model) of the CSI. The results show that gains of the proposed approach are mainly observed in the low SNR regime, at high traffic loads and at values of correlation coefficient above $\rho > 0.5$. The results also showed one more benefit, which is stable performance. The control of the transmissions of terminals based on channel state information allows NDMA protocol to remain stable for values of traffic load below the maximum channel nominal rate, which in the conventional NDMA protocol is lost due to imperfect presence terminal detection at the BS side. When the correlation coefficient that describes the accuracy of the CSI tends to one ($\rho \to 1$) the gains tend asymptotically to the maximum channel rate of one packet per time slot. This means that the potential gains will depend on the ability to obtain accurate (not outdated) channel state information for the terminals in the network.

APPENDIX I

A. Derivation of the CF of the instantaneous detection variable z_j in (14) conditional on $\hat{z}_j \geq \hat{\beta}$

The CF of the instantaneous detection variable z_j conditional on the estimated variable $\hat{z}_j$ being in the range $\hat{z}_j \geq$ can be obtained as in (13):

$$\Psi_{z_j|\hat{z}_j \geq \hat{\beta}}(i\omega) = \int_{\hat{\beta}}^{\infty} (1 - i\omega\tilde{\gamma})^{-1} e^{\frac{i\omega\rho^2 \hat{z}_j}{1-i\omega\tilde{\gamma}}} f_{\hat{z}_j}(\hat{z}_j) d\hat{z}_j. \quad (23)$$

By substituting the PDF of $\hat{z}_j$ given by $f_{\hat{z}_j}(\hat{z}_j) = \frac{1}{\breve{\gamma}} e^{-\frac{\hat{z}_j}{\breve{\gamma}}}$, where $\breve{\gamma} = J^2\gamma$ in the previous expression, and using the change of variable $\dot{\gamma} = \rho_c^2\breve{\gamma} + \tilde{\gamma})$ we obtain:

$$\Psi_{z_j|\hat{z}_j \geq \hat{\beta}}(i\omega) = \int_{\hat{\beta}}^{\infty} (1 - i\omega\tilde{\gamma})^{-1} e^{-y\frac{1-i\omega\dot{\gamma}}{\breve{\gamma}(1-i\omega\tilde{\gamma})}} \frac{1}{\breve{\gamma}} dy,$$

which after integration becomes:

$$\Psi_{z_j|\hat{z}_j \geq \hat{\beta}}(i\omega) = \frac{e^{-\frac{\beta_m(1-i\omega\dot{\gamma})}{\breve{\gamma}(1-i\omega\tilde{\gamma})}}}{1 - i\omega\dot{\gamma}}$$

which finalizes the derivation of (14)

B. Derivation of $\bar{W}(a,b,c,y)$ in (15)

Consider the Taylor series expansion of the exponential term in (14):

$$\frac{e^{a\frac{1-i\omega b}{1-i\omega c}}}{(1-i\omega b)} = \frac{1}{1-i\omega b} + \sum_{r=1}^{\infty} \frac{a^r}{r!} \frac{(1-i\omega b)^{r-1}}{(1-i\omega c)^r}. \quad (24)$$

Consider expansion of the second term in (24) using the binomial theorem as follows:

$$\frac{e^{a\frac{1-i\omega b}{1-i\omega c}}}{(1-i\omega b)} = \frac{1}{1-i\omega b} + \sum_{r=1}^{\infty} \frac{a^r}{r!} \frac{(1-i\omega b)^{r-1}}{(1-i\omega c)^r} =$$

$$\frac{1}{1-i\omega b} + \sum_{r=1}^{\infty} \frac{a^r}{r!} \sum_{l}^{r-1} \binom{r-1}{l} \frac{(-i\omega b)^l}{(1-i\omega c)^r}.$$

By using the properties of the Fourier transform analysis, the back-transform of this last expression is given by

$$W(a,b,c,y) =$$

$$\frac{1}{b} e^{-\frac{y}{b}} + \sum_{r=1}^{\infty} \frac{a^r}{r!} \sum_{l}^{r-1} \binom{r-1}{l} b^l \frac{d^l}{dy^l} \left(\frac{y^{r-1}}{c^r (r)!} e^{-\frac{y}{c}} \right)$$

$$= \sum_{r=1}^{\infty} \frac{a^r}{r!} \sum_{l}^{r-p} \binom{r-1}{l} b^l \sum_{k=0}^{l} \binom{l}{k} \frac{(-1)^l y^{r-1-l}}{c^{r+l} r!} e^{-\frac{y}{c}}. \quad (25)$$

The CCDF of this expression ($\bar{W} = \int_y^{\infty} W dy$) can be written as:

$$\bar{F}(a,b,c,y) = e^{-\frac{y}{b}} + \sum_{r=1}^{\infty} \frac{a^r}{r!} \sum_{l}^{r-1} \binom{r-1}{l} b^l$$

$$\times \sum_{k=0}^{l} \binom{l}{k} \sum_{s=0}^{r-1-l} \left(\frac{(-1)^l y^s (r+l)!}{c^{r+l} r! s!} e^{-\frac{y}{c}} \right). \quad (26)$$

which concludes the derivation of (15).

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