Kronecker Algebra for Static Analysis of Barriers in Ada

Robert Mittermayr, Johann Blieberger

Institute of Computer Aided Automation TU Vienna, Austria {robert,blieb}@auto.tuwien.ac.at

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- ullet matrices out of $\mathcal{M}=\{M=(m_{i,j})\,|\,m_{i,j}\in\mathcal{L}\}$ only.

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 - so-called *entry* node $n_e \in V$
- sets V and E constructed out of the elements of $\langle \mathcal{T}, \mathcal{S}, \mathcal{L} \rangle$.

Kronecker Product

Given an m-by-n matrix A and a p-by-q matrix B, their $Kronecker\ product$ $A\otimes B$ is an mp-by-nq block matrix defined by

$$A \otimes B = \begin{pmatrix} a_{1,1} \cdot B & \cdots & a_{1,n} \cdot B \\ \vdots & \ddots & \vdots \\ a_{m,1} \cdot B & \cdots & a_{m,n} \cdot B \end{pmatrix}.$$

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Given two automata, the Kronecker product synchronously executes them (lock-step).

Kronecker Sum

Given a matrix A of order m and a matrix B of order n, their Kronecker $sum\ A \oplus B$ is a matrix of order mn defined by

$$A \oplus B = A \otimes I_n + I_m \otimes B$$

where I_m and I_n denote identity matrices of order m and n, respectively.

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- Kronecker sum calculates all possible interleavings of two concurrently executing automata
- even if the automata contain conditionals and loops.

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Given an m-by-n matrix A and a p-by-q matrix B, we call $A \oslash_L B$ their selective Kronecker product. For all $I \in L \subseteq \mathcal{L}$ let

$$A \oslash_L B = (a_{i,j}) \oslash_L (b_{r,s}) = (c_{t,u})$$
, where

$$c_{(i-1)\cdot p+r,(j-1)\cdot q+s} = \begin{cases} I \\ 0 \end{cases}$$

if
$$a_{i,j} = b_{r,s} = I$$
, $I \in L$, otherwise.

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Selective Kronecker product ensures that, e.g., a semaphore p-call in the left operand is paired with the p-operation in the right operand and not with any other operation in the right operand. In practice, we usually constrain $L \subseteq \mathcal{L}_S$.

Filtered Matrix

We call M_L a *filtered matrix* and define it as a matrix of order o(M) containing entries of $L \subseteq \mathcal{L}$ of $M = (m_{i,j})$ and zeros elsewhere:

$$M_L = (m_{L;i,j})$$
, where $m_{L;i,j} = \begin{cases} m_{i,j} & \text{if } m_{i,j} \in L, \\ 0 & \text{otherwise.} \end{cases}$

System Model

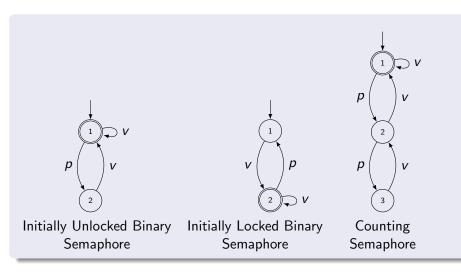
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$$P = T \oslash_{\mathcal{L}_{S}} S + T_{\mathcal{L}_{V}} \otimes I_{o(S)}.$$

Examples



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$$\begin{pmatrix}
0 & p & p & 0 \\
0 & 0 & 0 & v \\
0 & 0 & 0 & p \\
0 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
0 & p & p & 0 \\
0 & 0 & 0 & v \\
0 & 0 & 0 & p \\
0 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{pmatrix}
v & p \\
v & 0
\end{pmatrix} = \begin{pmatrix}
0 & 0 & 0 & p & 0 & p & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & v & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$
(a) CFG

self-deadlock at node 6

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- Static barriers have a statically fixed number of participating tasks/threads.
- The number of threads can vary at runtime for dynamic barriers.

```
package Ada.Synchronous_Barriers is
    pragma Preelaborate(Synchronous_Barriers);
    subtype Barrier_Limit is Positive range 1 .. implementation-defined;
    type Synchronous_Barrier (Release_Threshold : Barrier_Limit) is limited private;
    procedure Wait_For_Release (The_Barrier : in out Synchronous_Barrier;
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private
    -- not specified by the language
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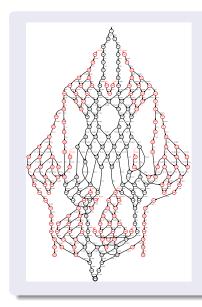
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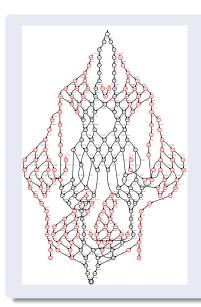
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In this paper: only static barriers

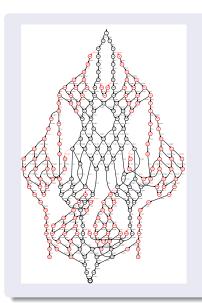
Implementation of Barriers

```
mutex.wait()
                            # ps
    count += 1
    if count == n:
        turnstile2.wait()
                           # pb2, lock the second
        turnstile.signal() # vb1, unlock the first
    else # empty
                            # T1.a: T2.e
mutex.signal()
                            # vs
                           # pb1, first turnstile
turnstile.wait()
turnstile.signal()
                            # vb1
                            # T1.b: T2.f
# critical point
mutex.wait()
                            # ps
    count = 1
                            # d
    if count == 0:
        turnstile.wait()
                            # pb1, lock the first
        turnstile2.signal() # vb2, unlock the second
                           # T1.c; T2.g
    else # empty
mutex.signal()
                            # vs
                            # pb2, second turnstile
turnstile2.wait()
turnstile2.signal()
                            # vb2
```

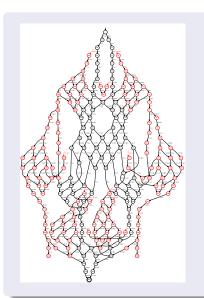




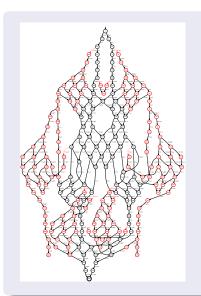
 The CPG contains potential deadlock nodes 681, 761, 1774, 1790, 1961 and 2030.



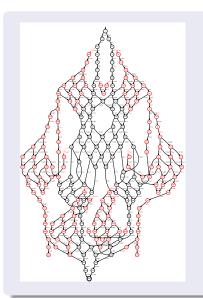
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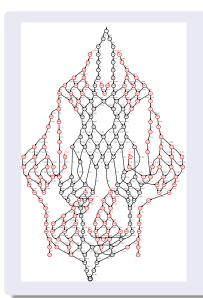
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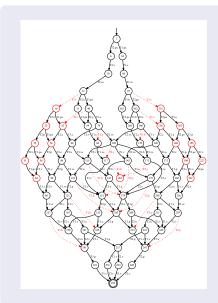


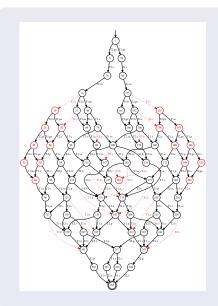
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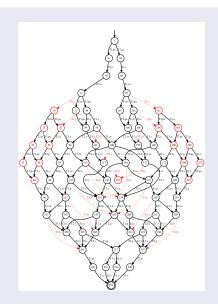
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- Implementation using three semaphores is correct.
- Advanced approaches like symbolic analysis are needed.

Implementation of Barriers

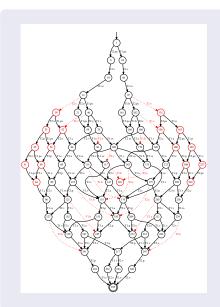




 Again there are dead paths (the corresponding edges are dotted).

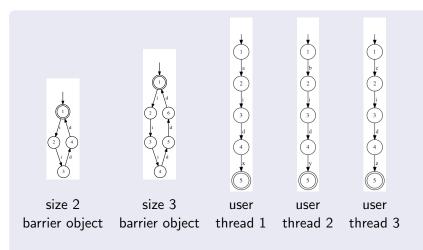


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- Deadlock node (node 181).

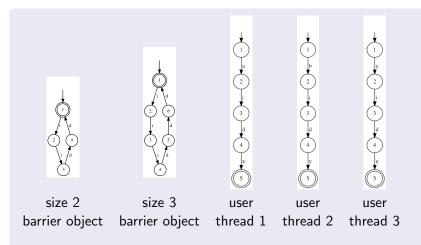


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- Deadlock node (node 181).
- Paths to node 181 are dead paths.

Barrier Synchronization Object

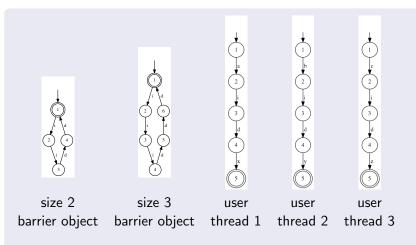


Barrier Synchronization Object



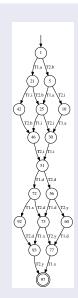
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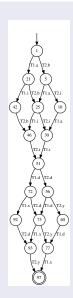


- models "semantics of barriers" instead of "implementation"
- cannot verify implementation

Barrier Synchronization Object – Example: Two threads

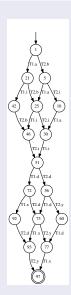


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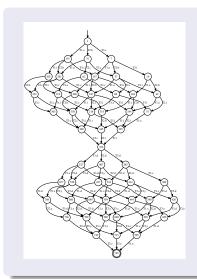
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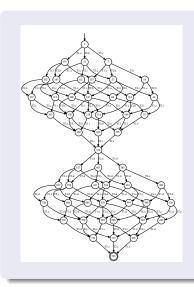


- free of deadlocks
- no need for value sensitive analysis

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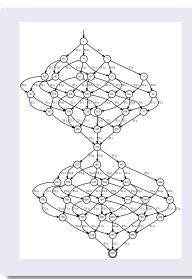


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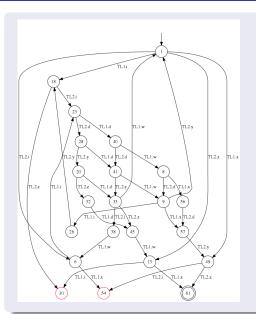


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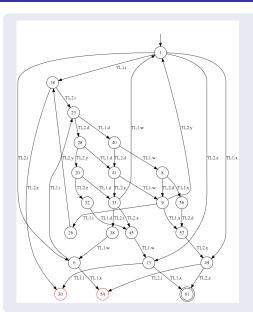
Barrier Synchronization Object – Example: Three threads



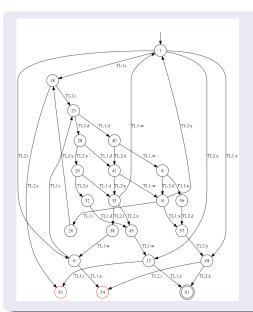
- free of deadlocks
- no need for value sensitive analysis



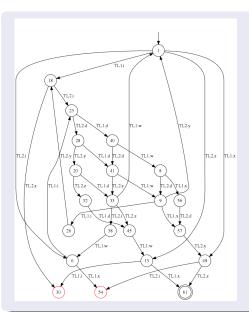
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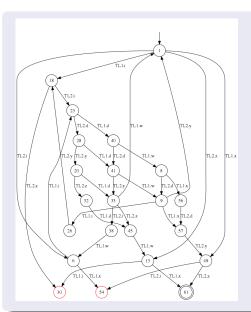
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- In the simplest case, only lower and upper bounds of for-loops have to be compared.

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- Need advanced techniques for programs containing loops or conditional statements.